Solving proportional and analogical problems

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Abstract. The purpose of the present study was twofold. First to examine the relationship between proportional (arithmetic) and analogical (verbal) problems and second to examine the breath of strategies used while solving them by Cypriot students in fifth and sixth primary grade. The results showed that students confront in a different way proportional and analogical problems. Concerning the strategies used, fifth grade students seem to prefer finding the factor of change while sixth grade students prefer to use the rule of three method. The results also reveal that a great percentage of students of both ages have not a complete perception of the relationships which condition an analogy. The application of the implicative statistical analysis of R. Gras and more general the CHIC software give a clear idea of the relations between proportional and analogical problems according to students' replies.

1 Introduction

Proportion is a fundamental concept in math curriculum, as it has been proven to promote the development of mathematical thinking (Confrey & Smith, 1995; Nabors, 2003). It appears at the first grades of primary school mathematics in verbal multiplication and division problems. Gradually, it develops in situations involving fraction equivalence or fraction comparison to result as a form of basic knowledge for the development of algebraic relationships, trigonometry and probability theory (Papageorgiou & Christou, 1999).

Despite the fact that proportion appears early within the mathematics curriculum, a substantial amount of research has shown that it constitutes a difficult concept for the students (Nabors, 2003). Post, Behr and Lesh (1988) state that only a small number of high school students use proportional thinking in a right way, something that has been also observed to happen in tertiary education (Lawton, 1993). In parallel, there is evidence that a great amount of population does not acquire proportional thinking sufficiently (Hoffer, 1988).

Proportion consists of a second-order relationship, which includes an equivalence relationship between two ratios (Christou & Philippou, 2002), e.g. a/b=c/d. Proportion comprises of four elements, whereas the kind of relationship among them determinates the kind of strategy to be used for solving problems (Lamon, 1994). These relationships are divided into two categories, depending on the type of comparisons: "within" relationships, that is relationships between quantities of the same kind and "between" relationships, namely relationships between corresponding quantities of different kind (Kaput & West, 1994). For example, at the problem "If 3 kilos of potatoes cost 90 cents, how much do I have to pay for 12 kilos?", there are two metric spaces, one of kilos and the other of cents. "Within" relationships refer to comparison of same kind quantities (e.g. kilos with kilos) while "between" relationships apply when comparing quantities of different kinds (e.g. kilos and money).

Ben-Chaim et al. (1998) refer to three kinds of tasks which demand proportional reasoning: (a) missing value problems, where the three elements of the analogy are known and the fourth one is missing, (b) numerical comparison problems, where both ratios of the analogy are given and the student has to compare them and (c) qualitative prediction and comparison problems, which demand for comparisons not based in specific arithmetic values.

Due to the importance of proportional reasoning, a substantial amount of research has investigated both correct and incorrect strategies used by students in their attempt to solve unknown quantity proportional problems (Christou & Philippou, 2002; Karplus, Pulos & Stage, 1983; Kaput & West, 1994; Tourniaire & Pulos, 1985). Proportional problem solving methods can be distinguished into two categories on the basis of their structure: strategies with multiplicative structure and strategies with additive structure (Tourniaire & Pulos, 1985). Strategies which belong in the first category, involve unit-rate, factor of change, rule of three and equivalent fractions (Bart, Post, Behr, & Lesh, 1994). The second category includes building-up methods, which are a more informal method of proportional reasoning (Christou & Philippou, 2002), including equivalent class and pair generation (Bart et al., 1994). The use of a certain strategy depends on the kind of the problem and the relationships between its numerical variables (Christou & Philippou, 2002; Karplus, Pulos & Stage, 1983; Tourniaire & Pulos, 1985).

The most prevalent approach for solving missing value proportional problems found in mathematics textbooks in Cyprus is the "cross-multiply" method (Christou & Philippou, 2002). "Cross-multiply" method is a mnemonic rule for solving proportional problems and opposes to informally-based student solutions (Kaput & West, 1994).

Based on the literature, the additive method is the most common incorrect strategy regarding proportions (Inhelder & Piaget, 1958; Hart, 1984). Misailidou and Williams (2003) have constructed a proportional thinking diagnostic assessment tool which assessed among others students' tendency to apply additive methods.

A variety of models (Droujkova & Berenson, 2003) show direct connection between proportional reasoning in mathematics and psychology. In mathematics the term used is proportional reasoning, while the corresponding term used in psychology is analogical reasoning. This distinction between the two terms has been also followed in the present research. Piaget and Campbell (2001) claim that: « analogies… are a sort of qualitative proportions. They are relations among relations" (p. 139). Analogical reasoning includes important structural relationships and connections among situations or ideas (English & Sharry, 1996).

Analogies can be formed in a variety of ways and involve various semantic relationships between concepts in various levels (Hoffman, 1998). These relationships vary from concepts connected by superficial characteristics (e.g. words with the same number of letters), similarity and difference relations, to concepts related with higher level characteristics (g.e. place, function, change of state) (Sternberg, 1977). A typical form of an analogy can be presented as A : B :: C : .., where the relation among A and B concepts of the first pair applies or is transferred to C and D concepts of the second pair. The unknown element can be any of the four concepts or one pair. An example of an analogy in which the terms are related according to their place is the following: Pharmacy: Drugs ::Green-grocery: Fruit (Hoffman, 1998).

There is evidence showing the relationship between analogical reasoning and learning (Vosniadou, 1989), teaching (Alexander, Willson, White & Fuqua, 1987) and intelligence or creativity (Marr & Sternberg, 1986). Nevertheless there have not been made many efforts to investigate the existence of a relationship between proportional and analogical problems. Therefore, the purpose of the present study is to investigate the existence of a relationship between proportional and analogical problems and to examine the variety of methods used by Cypriot 5th and 6th grade primary school students in proportional problems.

2 Method

Data were collected from 301 students attending 5th and 6th grade in primary schools in Cyprus. Specifically, the sample of the study consisted of 139 students of 5th grade (10 years old) and 62 students of 6th grade (11 years old).

A written test was used to collect the data, which was given to all 301 students. The test consisted of 8 problems (see Appendix), four of which were proportional (unknown quantity) and four analogical problems. Proportional problems were categorized according to the relationship among their numerical variables. The first problem included "within" and "between" multiplicative relationships. The second problem dealt with whole number "between" multiplicative relationships. This problem comprised a variation of the "onion soup" problem used in a study by Hart, Brown and Küchemann (1984). The fourth problem involved whole number "within" multiplicative relationships, while the third problem did not include whole number multiplicative relationships among its terms.

In order to solve each problem, students had to compare ratios resulting from the numerical variables using whole number "between" and "within" relationships or other algorithmic procedures. Students were also asked to explain the method they used in solving each problem of the test.

The purpose of the study was not to obligate students to execute complex algorithmic procedures but to examine the methods used. Therefore, numbers used in proportional problems were purposively relatively small.

Analogical problems aimed to examine the way words are connected to each other. In each problem, two pairs of words were presented and students had to find the relationship between the words of the first pair, in order to complete the missing term(s) in the second pair. For each missing word, three alternative words were given. In the first two analogical problems, students had to fill the second word of the second pair while at the next two problems they were asked to find the whole pair.

The results concerning students' responses in each proportional problem were codified as follows: Correct or incorrect response (Pa), correct or incorrect explanation (Pe) and the type of students' solution strategies (Sa), (Sb), (Sc) (Sd) or (Se). These numbers correspond to: Rule of three (Sa), Factor of change (Sb), Unit-rate (Sc), Additive (incorrect) strategy (Sd) and building-up method (Se).

Students' responses in each analogical problem of the test were codified as correct or incorrect (A).

For instance, the variable Pa1 refers to the solution of the first proportional problem; the variable Sa2 indicates the use of the rule of three at the second proportional problem, whereas the variable A3 refers to the completion of the third analogical problem. All variables were codified as 0 and 1. Therefore, each correct solution was assigned the score of 1, while each wrong solution was given the score of 0. In a similar way, the use of a particular strategy was codified as 1 and the non use as 0.

For the analysis and processing of the data, we used the statistical package of SPSS to perform t-tests for independent groups, as well as the computer software CHIC to carry out the hierarchical clustering of variables (Lerman, 1981) and the implicative statistical analysis (Gras, Briand & Peter, 1996; Gras, Peter & Philippe, 1997). From the two methods of analysis in CHIC, the similarity and implicative diagrams were derived. The similarity diagram represents groups of variables which are based on the similarity of students' responses to the test's tasks. These similarity groups are hierarchically arranged based on the strength of their homogeneity. The implicative diagram involves relations between students' responses, which show whether success on a task implies success on another task.

3 Results

Cronbach's alpha reliability index was satisfactory (a=0.656). Table 1 presents an overview of students' performance in proportional and analogical problems in both grades. In particular, 6th grade students outperformed 5th grade students in all problems, both in proportional problems ($\overline{X}$ 5th=0.42, $\overline{X}$ 6th =0.66) and analogical problems ($\overline{X}$ 5th =0.45, $\overline{X}$ 6th =0.58). These differences were statistically significant both at the proportional ($t=-6.57$; $p<0.01$) and the analogical problems ($t=-4.06$; $p<0.01$). The results suggest that students' performance on proportional and analogical tasks increases significantly from grade 5 to grade 6.

The table also reveals that among proportional problems the most difficult problem for both grades was the third problem, whereas the first problem was the easiest to solve. Concerning analogical problems, the second problem was the most difficult for both grades, whereas the first problem was again the easiest to solve.

	Problems			
	Proportional		Analogical	
Problem	5th grade	6th grade	5th grade	6th grade
1	0,80	0,84	0,78	0,81
2	0,30	0,62	0,27	0,35
3	0,26	0,53	0,35	0,54
4	0,32	0,65	0,38	0,61
Total	0,42	0,66	0,45	0,58

TAB. 1 - Students' mean performance in all test items

The most commonly used methods by students in solving proportional problems were the rule of three, finding the factor of change, unit-rate and building-up (Table 2). One of the incorrect strategies used by students is the additive strategy. Other incorrect strategies were not categorized due to low occurrences of use. They were instead grouped in a category named "Other incorrect method". Absence of a method was categorized separately, and involved both responses without appearance of a particular method and empty responses.

Rule of three is used by 6th graders in a greater percentage in all proportional problems (5th grade: 3%; 6th grade: 25%) whereas the remaining methods are used almost in the same percentage by students in both grades. Finding the factor of change appears to be the most frequently used method among students in both grades. This method was mainly used in the first proportional task, where the whole number relationships that were involved helped both 5th and 6th graders in finding the factor of change.

Additive incorrect method was observed in a relatively large percentage (27%) in the third proportional problem among 5th graders. This was probably due to the increased difficulty of this problem, also shown by the low performance on it. The majority of 6th grade students who solved the particular problem used the rule of three.

Problem	Rule of three (Sa)/%		Factor of change (Sd)/%		Unit-rate (Sc)/%		Building-up (Se)/%		Additive method (Sd)/%		Other incorrect method/%		Absence of method/%	
	Methods													
	5th	6th	5th	6th	5th	6th	5th	6th	5th	6th	5th	6th	5th	6th
1	2	19	68	62	-	2	-	1	1	2	12	6	17	8
2	1	23	11	12	9	9	7	15	-	-	42	22	30	19
3	4	30	6	10	14	15	1	1	27	9	19	10	29	25
4	4	26	26	35	2	2	1	1	4	2	24	9	39	25
Total	3	25	28	30	6	7	2	5	8	3	24	25	29	19

TAB. 2 - Students' percentages of method use in the proportional test items

Figure 1 presents the similarity relations among 5th grade students' responses to all test items and their explanations in the proportional problems. Two clusters of similarity are identified: Cluster A and B. In Cluster A the responses and explanations of proportional problems 1 and 4 are grouped together with the solution to the third analogical problem. In Cluster B the responses and explanations to proportional problems 2 and 3 are grouped with the answers to the analogical problems 1, 2 and 4. This indicates that first cluster problems where handled in a different way than second cluster items by 5th grade students. A possible explanation is that proportional problems 1 and 4 included "within" whole number multiplicative relationships, and therefore students solved them using this structure. The fact that the proportional problems 2 and 3 did not include "within" multiplicative relationships among their terms differentiated students' solutions.

However, between the two clusters, it appears to be a systematic way in which the items were handled by the students, concerning the provision of an explanation in proportional problems. In particular, in Cluster A strong similarity relations are established between the responses and explanations to the first proportional problem and to the fourth proportional problem respectively. Moreover, in the second cluster of similarity, there are strong similarity relations between the responses and explanations to the second and third proportional problem respectively. These relations suggest that there is consistency in the ways students respond and explain their answers to each proportional problem. In other words, students who give correct responses to the proportional problems can give appropriate explanations for them and vice-versa. In each cluster weak similarity relations appear between the responses to the proportional problems and the answers to the analogical items. This suggests that students respond to the proportional problems by activating different processes relatively to the analogical problems, the majority of which (A1, A2 and A4) seem to be handled in a similar way.

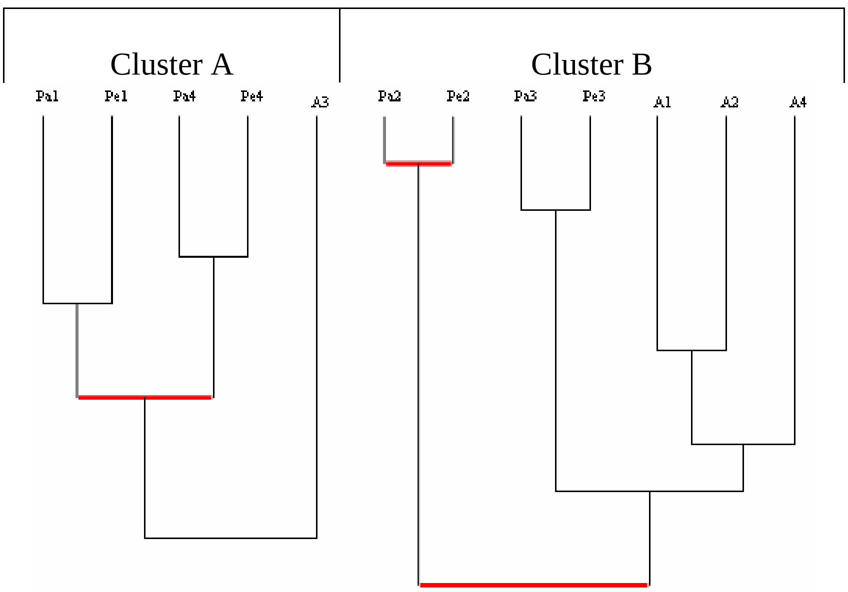

FIG. 1 - Similarity diagram of the responses and explanations in proportional problems and responses to analogical problems in 5th grade

Figure 2 is the implicative diagram involving the responses of 5th graders in all test items, their explanations and methods used in proportional problems. As far as analogical problems are concerned, only problems 1 and 3 are related to the third proportional problem. Provision of an explanation in any proportional problem, implies a correct answer in each of these (Pei →Pai). This means that most of the students, who explained their solution process, provided a correct answer to the proportional problems. In every proportional problem, except for the second one, the use of factor of change as a solution strategy implies a correct answer in each problem (Sbi→Pai). In addition to the factor of change method, the use of the rule of three and unit rate implies correct solution to the third problem, while the use of the rule of three implies success to the fourth problem as well. In the case of the second proportional problem, only the application of the unit-rate implies giving a correct answer (Sc2→Pa2). Using additive and therefore incorrect method in the fourth proportional problem implies its use also for dealing with the third problem. The easiest tasks for 5th grade students were the first analogical task and the first proportional one.

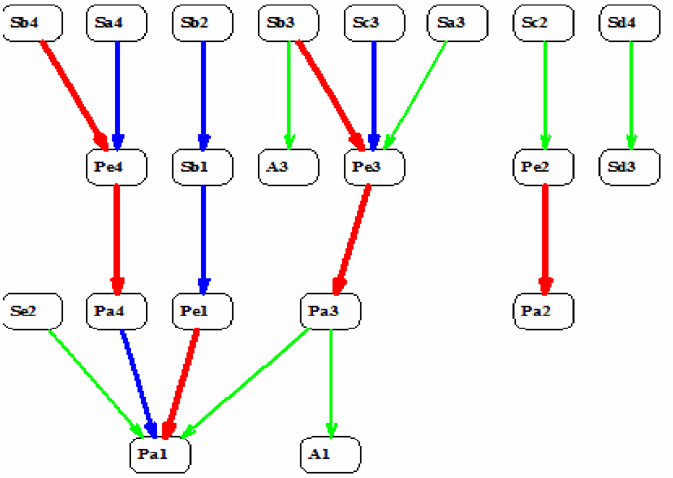

FIG. 2 - Implicative diagram among 5th grade students' responses in all test items, their explanations and methods used in proportional problems

Figure 3 presents similarity relations among 6th grade students' responses in all test items and the methods used in solving proportional problems. Based on students' responses, two similarity clusters are formed: Cluster A and B. In Cluster A the responses and explanations in all proportional problems and the correct answer in the second analogical problem are grouped together. Cluster B is completely independent from the first cluster. It consists of the analogical problems 1, 3 and 4. This indicates that 6th grade students handled differently proportional and analogical problems, except for the second analogical problem, but with a very weak similarity relation with the second proportional problem. This finding is in line with 5th grade students' differentiated behavior towards the two types of tasks, as shown in Figure 1.

The structure of Cluster A reveals a systematic way in which sixth graders handled the proportional problems, which is also similar to the fifth graders' behavior. The correct solution to each proportional problem is paired with the explanation given for its solution process. Cluster A is divided into two similarity groups: Group 1 and 2. Group 1 consists of the solutions and explanations in proportional problems 1, 3 and 4, whereas Group 2 includes the response and explanation in the second proportional problem and the correct solution at the second analogical problem. Group 1 is divided into two subgroups (a and b), which are also linked together. The first subgroup (a) consists of the solution and explanation in the first proportional problem and the solution and explanation in the fourth proportional problem. The fact that both problems included "within" multiplicative relationships between their variables may provide an interpretation of the strong consistency shown by the students in their solution. The second subgroup (b) involves the solution and explanation in the third proportional problem. Despite third proportional problem's different type of numerical structure, which did not involve "within" or "between" whole number multiplicative relationships, students approached it in a relatively similar way to the problems 1 and 4. Concerning the second group of similarity (Group 2), there is strong similarity between the correct answer and the explanation of the second proportional problem. The fact that this group is linked to Group 1, which involved the responses to the other three problems, suggests that 6th grade students did not handle the second problem in a completely distinct way relatively to the other problems, despite the fact that it did not include “within”, but “between”, multiplicative relationships. These findings in combination with the remarks on the similarity diagram of 5th grade students suggest that the proportional problems’ inclusion of “within” or “between” whole number multiplicative relationships exerted stronger effect on 5th grade students’ performance rather than on 6th grade students’ performance.

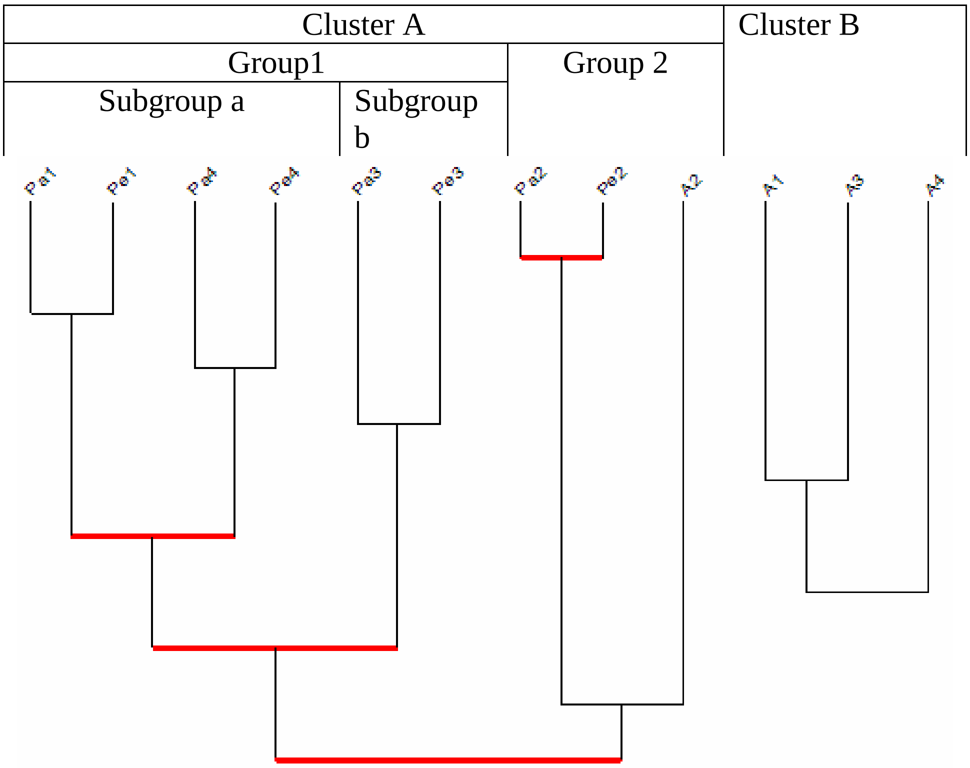

FIG. 3 - Similarity diagram of the responses and explanations in proportional problems and responses to analogical problems in 6th grade

Figure 4 is the implicative diagram involving the responses of 6th graders in all test items, their explanations and methods used in proportional problems. Using the method of three in the first two proportional problems implies the use of the same method in problem 4 and subsequently also in problem 3. Using this method though, does not imply directly the provision of a correct solution and explanation in proportional problems, except for the third one. Sixth grade students seemed to employ the rule of three more flexibly and consistently over all of the proportional problems than fifth grade students. The use of different strategies implies correct responses and explanations in the proportional problems. On one hand, using mainly one method and particularly the factor of change implies correct solutions to problems 1 and 4, which included “within” whole number multiplicative relationships. On the other hand, using a variety of strategies implies a correct response and/or explanation for the solution process in problems 2 and 3, which did not involve “within” whole number multiplicative relationships. Specifically, the use of the factor of change, the unit-rate and the building-up method entails successful solution in the second problem, while using the rule of three, the factor of change and the unit-rate implies correct response or explanation in the third problem. A hypothetical explanation for the difference in the plurality of strategies yielding correct solutions to the problems is that the increased difficulty of the numerical structure of the problems 2 and 3 relatively to problems 1 and 4 induced a number of students to look for and try out alternative methods other than the factor of change. This behavior was probably a consequence of their unsuccessful initial attempts to employ the (most “favorable”) factor of change method for solving the problems. Similar findings occurred in the implicative diagram of fifth graders’ responses (Figure 2), regarding the third proportional problem. Using three different methods (unit-rate, factor of change and rule of three) entailed correct explanation and response to the problem. However, this was not the case in the second proportional problem, as mostly students who used the unit-rate method provided correct solution and explanation to the problem. This finding as well as 5th grade students' lower performance to the problem suggests their decreased flexibility in successfully using different strategies for providing a solution, relatively to 6th grade students.

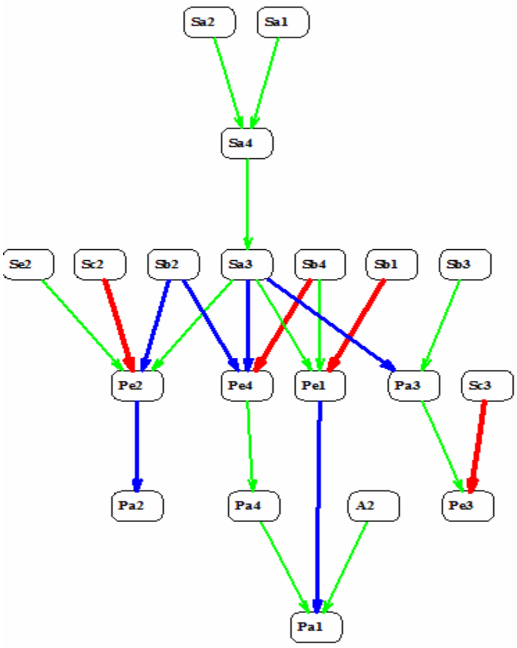

FIG. 4 - Implicative diagram among 6th grade students' responses in all test items, their explanations and methods used in proportional problems

Providing a correct explanation for the solution process of each proportional problem implies a correct answer (Pei →Pai), except for the third problem. While the use of the rule of three and the factor of change implies successful solution of the third problem, only the use of the unit-rate entails an adequate explanation of the solution process. Students who used the unit-rate found it easier to explain their solution relatively to the students who used other, but also successful, methods in the particular problem, which did not involve whole number multiplicative relationships among its terms. Concerning analogical problems, only the second problem appears in the implicative diagram and implies the correct solution of the first proportional problem.

4 Conclusions

The present study aims to investigate the existence of a relationship between proportional and analogical problems and to examine the variety of methods used by Cypriot 5th and 6th grade primary school students in proportional problems.

The analysis of the data reveals that 6th grade students' performance in both proportional and analogical problems was clearly higher compared to 5th grade students' performance. These results were expected, due to the increased amount of experience of 6th grade students when compared with 5th grade students. This finding is in line with the findings of Christou and Philippou (2001), according to which the impact of school teaching has a decisive role in the development of the concept of proportion.

Concerning the solution methods used, 5th and 6th grade students preferred to use factor of change in order to solve proportional problems. At the same time, 6th grade students applied in many cases the rule of three, unlike 5th grade students. This finding may be due to the fact that this method is formally taught in 6th grade. However, in most cases strategies other than the "rule of three" yielded successful responses and explanations for the solution processes to the problems. This finding gives an indication of the mechanical way of employing this method in proportional problems by a number of students. Moreover, the large percentages (about 25%) of using an erroneous strategy reveal that students have not gained a thorough understanding of the relations and conditions of a proportion.

The numerical structure of the proportional problems was found to influence students' consistency and strategies. This influence differed as a function of school grade. Students dealt consistently with problems involving "within" whole number multiplicative relationships. However, they approached these problems differently from the problems involving "between" whole number multiplicative relationships or problems not involving whole number multiplicative relationships. This phenomenon and therefore the impact exerted by the problems' numerical structure on students' consistency was stronger in the case of 5th grade students. It is suggested that sixth grade students' were more flexible in dealing with different proportional situations despite their surface dissimilarities (relations of numbers), probably because of their greater learning experience with proportional situations in school. Furthermore, the numerical structure of the problems was found to stimulate different strategies among the students. The inclusion of "within" whole number multiplicative relationships yielded mainly the application of the factor of change method, while the inclusion of "between" whole number multiplicative relationships yielded a plurality of other strategies in addition to the factor of change, such as the use of the unit-rate, the rule of three and the building-up method. This finding was found to apply mostly among sixth grade students, whose flexibility in using different strategies (especially when realizing that a particular strategy was not efficient) helped them in attaining more correct solutions to the problems relatively to fifth grade students.

Finally, students' responses reveal that both 5th and 6th grade students handle differently proportional and analogical problems. Therefore, a subsequent research could further investigate the relationship between students' responses in proportional and analogical problems.

It is crucial for students to get familiar with a breadth of both proportional and analogical problems, in order to be able to choose the appropriate method and realize the limitations and advantages of each strategy in solving a proportional problem. Additionally, tasks like those used in the test could and should be used by educators in order to understand the problems that their students confront in proportional and analogical problems. In this way, teachers could shape appropriately their teaching in order to help students to better acquire the concept of proportion.

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Résumé

Le but de ce travail consiste en la recherche de l'existence de relations entre les problèmes numériques et verbaux portant sur les proportions et l'examen de l'ampleur des stratégies des élèves de la 5ème et 6ème de l´école primaire de Chypre. Les résultats de l'enquête montrent que les problèmes numériques et verbaux sur les proportions sont traités différemment par les élèves. En ce qui concerne les stratégies pour la résolution de problèmes proportionnels numériques, les élèves de 5ème semblent préférer la découverte du facteur de changement, tandis que les élèves de 6ème la règle de trois. De plus, on constate un grand pourcentage des élèves des deux classes qui n'ont pas une perception intégrée des relations qui gouvernent une proportion. L´application de l´analyse implicative de R. Gras et spécifiquement du logiciel CHIC donne une idée claire, à partir des réponses des élèves, des relations entre problèmes mathématiques de proportionnalité et relations d´analogie verbale.

Appendix

The tasks

Proportional problems

1. There are two dogs in the camping area, namely Skinny and Max. The dogs eat dog food cans according to their weight. Skinny weighs 3kg and eats 9 dog food cans. Max weighs 6kg. How many dog food cans does Max eat?

2. The cook of the camping will prepare pancakes for the children, using the following recipe which is for 12 pancakes:

1 cup of flour
2 eggs
1 cup of milk
1 spoon of oil

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2 spoons of sugar

1 small spoon of vanilla

If the cook intends to make 54 pancakes, how many eggs will he need?

3. The children in the camping have been divided into groups and participate to competitions for the "Young Walkers" award. Mary and Alex observe the table of the points of each team. The Yellow Team won in 6 games and got 15 points. The Red Team won in 4 games. How many points did the Red Team get?

4. In the forest exploration children use a map. The scale on this map is 3 to 80 (that is 3cm on the map correspond to 80cm in reality). On the map there is a bridge 12cm long. How long is the bridge in reality?

Analogical tasks

1. Camper: Tent :: Bird: _______________

Cave, Nest, Cage

2. Sheep: Fleece:: Chicken: _______________

Eggs, Feathers, Meat

3. Bed: Sleep:: _________________:_________________

Paper	Food
Table	Rain
Water	Book

4. Bread: Knife:: _____________:_____________

Paper	Ink
Sheet	Scissors
Wood	Razor